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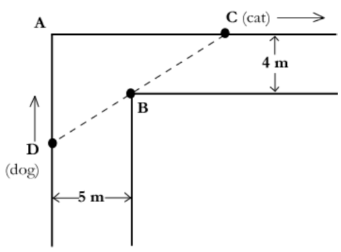
MAM1020S **Tutorial 10** **August 2017**

1. Air is being pumped into a spherical ballon at a rate of 20 in^3/min How fast is the radius of the balloon increasing when the radius is 6 in?
2. A 25-foot ladder is leaning against a vertical wall. The floor is slightly slippery and the foot of the ladder slips away from the wall at a rate of 0.2 in/sec. How fast is the top of the ladder sliding down the wall when the top is 20 feet above the floor?
3. a conical water tank with vertex down has a radius of 10 ft at the top of is 24 ft high. If water flows out of the tank at a rate of 20 ft^3/min, how fast is the depth of the water decreasing when the water is 16 ft deep?
4. On a dark night, a thin man 6 feet tall walks away from a lamp post 24 feet high at the rate of 5 mph. How fast is the end of his shadow moving? How fast is the shadow lengthening?
5. Ship A is 32 miles north of ship B and is sailing due south at 16 mph. At what rate is the distance between them changing at the end of 1 hour? Is the distance increasing or decreasing?
6. Two sides of a triangle have lengths of 12 m and 15 m. The angle between them is increasing at a rate of 2 degree per minute. How fast is the area of the triangle increasing when the angle between the sides of fixed length is 60 degree?
7. The volume of a cube is changing at the constant rate of 75 cubic cm/min.
   (a) Find the rate of change of an edge of the cube when the length of the edge is 5 cm.
   (b) Find also the rate of change of the surface area when the surface area is 24 square cm.
8. The edges of a right-angles triangle are changing but the perimeter is fixed 40 cm. When the hypotenuse is changing at a rate of $7cm/min$ and the edges are $8cm$, $15cm$, $17cm$, find the rates of change of the other edges at this instant.
9. A basin has the shape of an inverted cone with altitude 100 cm and radius at the top of 50 cm. Water poured into the basin at constant rate of $40cm/minute$. At the instant when the volume of water in the basin is 486π cubic centimetres, find the rate at which the level of water is rising.
10. A northbound ship leaves harbour at 12 noon with a speed of 7.5 knots and a westbound ship leaves the same harbour at 2 p.m. with a speed of $8knots$. How fast are the ships separating at 4 $p.m.$?
11. A man is running over a bridge at a rate of 5 metres per second while a boat passes under the bridge and immediately below him at a rate of 1 metre per second. The boat's course is at right angles to the man's and 6 metres below it. How fast is the distance the between the man and the boat separating 2 seconds later?
12. In the diagram shown the corridors are 4 m and 5 m wide. A cat is moving ti the righr at $10m/sec$. At the instant when a dog D is 12 meter from A, how fast must the dog run in order to keep the cat in sight?



13. An ice cube with a 10 cm side is melting, so that its volume decreases 12 cm^3/min. How fast is the surface area changing?
14. The height of a right circular cone is increasing at a rate of 1.5 cm/sec, while its volume is increasing at a rate of 2 $cm3/sec$. How fast is the radius of the base of the cone changing when the radius is $6cm$ and the height is $8cm$?

15. Larry and Moe have each bought recently a pickup truck and they meet to show them off to each other. As they leave, Larry heads West with a constant acceleration of $2m/sec$, while Moe looks at him for 3 seconds and then heads North-East also with a constant acceleration of $2m/sec$. After 10 more seconds, how far is Larry from the departure point? And how fast is the distance between the two friends changing?

Do the following steps for each functions f bellow:

1. Compute the domain of definition of f, if it is not given.
2. Find the intercepts.
3. Is there any symmetry: even/odd?
4. Compute the limits at any points where it is necessary as $-\infty$, ∞ (if in the domain), and any extreme point that is not in the domain of definition. (for instance $f(x) = 1/x$, its domain of definition is $(-\infty, 0) \cup (0, \infty)$ then you would compute the limit and $-\infty$, ∞, 0^+ and 0^-)
5. Deduce from the previous question the asymptotes if any.
6. Compute the derivative and decide about its sign. To deduce where f is increasing or decreasing.
7. Find the critical number for the function (all of them that is x such that $f'(x) = 0$ and $f'(x)$ does not exist.)
8. Find the local minimum/maximum and the x values that permit to reach them. Decide if they are global maximum/minimum. Justify.
9. Study the concavity of the function after computing the second derivative of f and deciding it sign. Give the intervals where it is concave downward and upward.
10. Give all the inflection points for your curve.
11. Compute a few extra points if needed and sketch the graph of your function.

Here are the functions:

1. $f(x) = xe^x$
2. $f(x) = xe^{-2x}$
3. $f(x) = x^2e^{-2x}$
4. $f(x) = x^2e^{2x}$
5. $f(x) = xe^{x^2}$
6. $f(x) = e^{-1/x^2}$
7. $f(x) = xln(x)$
8. $f(x) = x^2ln(x)$
9. $f(x) = \frac{ln(x)}{x^2}$
10. $f(x) = \frac{ln(x)}{x^{1/2}}$
11. $f(x) = \frac{x^2}{\sqrt{x+1}}$
12. $f(x) = ln(4 - x^2)$
13. $f(x) = \frac{x^2+7x+3}{x^2}$
14. $f(x) = \frac{x^2(x+1)^3}{(x-2)^2(x-4)^4}$
15. $f(x) = sin(x + sin(2x))$
16. $f(x) = \frac{1}{x^2+2x+c}$ (compare the graph when c varies, which properties do they have in common and which differ.)
17. $f(x) = x^3 - 3x^2 + 5$
18. $f(x) = \frac{x^2+5x+1}{x^2}$
19. $f(x) = x^5 - 5x + 1$
20. $f(x) = (x + 4)^3(x - 2)$
21. $f(x) = 3x^2(x^2 - 2)$
22. $f(x) = x^3 - 3x^2 + 3$
23. $f(x) = 2x^4 - x^2$
24. $f(x) = \frac{x^2+4}{x^2-4}$
25. $f(x) = \frac{x(2-x)}{(x-1)^2}$.
26. $f(x) = \frac{x}{(x+1)^2}$
27. $f(x) = \frac{3(x^2+1)}{x^2-9}$
28. $f(x) = \frac{9x}{(3x+1)^2}$
29. $f(x) = x^{1/3}(x + 8)$
30. $f(x) = \frac{e^x}{x}$.